SEMESTER-IV PAPER-V MODERN PHYSICS

UNIT-1 ATOMIC AND MOLECULAR PHYSICS

- **Vector atom model**:

To overcome the drawback of Bohr and Sommerfeld model, a new model known as vector atom model was proposed by Uhlenbeck and Goudsmit.

The two concepts of vector atom model are

(1) The concept of quantization of direction i.e., space quantization

(2) The concept of spinning electrons.

(1) Space quantization: We know that the electron moving around the nucleus in an atom is equivalent to a magnetic dipole. When the atom is placed in an external magnetic field B, the electron orbit precess about the field direction. Now the angular momentum vector P_l traces a cone around B. Let B is along z – axis. The component of P_l in the field direction is given by

$L_z = P_l \cos\theta \Rightarrow \cos\theta = \frac{L_z}{P_l}$

According to quantum mechanics

$P_l = \frac{lh}{2\pi}$ where $l -$ Orbital quantum number

$L_z = \frac{m_l h}{2\pi}$ $m_l -$ Magnetic orbital quantum number.

$\cos\theta = \frac{m_l}{l}$, $-1 \leq \cos\theta \leq 1$

$m_l = -l$ to $+l$

∴ for each value of l there will be $(2l+1)$ possible values of m_l.

So angular momentum vector P_l can have $(2l+1)$ discrete orientations of atom in space is known as space quantization.

Ex: consider the space quantization vector P_l corresponding to $l = 2$,

$m_l = -2, -1, 0, +1, +2$

$L_z = 2\hbar, \hbar, 0, -\hbar, -2\hbar$

$\cos\theta = \frac{m_l}{l} = \frac{m_l}{\sqrt{l(l+1)}}$

$\cos\theta = \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, 0, -\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}$

$\theta = 35^0, 66^0, 90^0, 114^0, 145^0$

(2) Spinning electron:

It was observed that in the spectra of alkali metals, all the lines were doublets. Bohr theory as well as Sommerfeld theory could not explain the nature of spectral lines. In order to explain the observed behaviour Uhlenbeck and Goudsmit postulated that the electron not only revolve round the nucleus but it spin around its own axis. Thus the

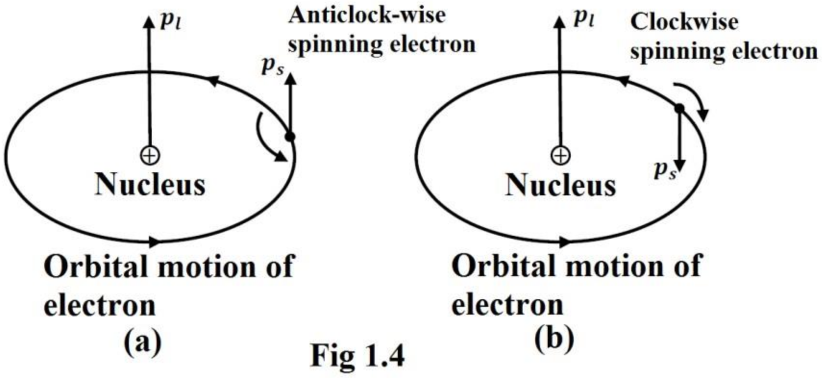


Fig 1.4

electron posses spin motion as well as orbital motion. The spin angular momentum of electron is given by

$P_s = \frac{sh}{2\pi}$ where $s = \frac{1}{2}$

When magnetic field is applied, P_s can have only two orientations.

i.e., $m_s = s\cos\theta$ or $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$

* The quantities that determine the state of the atom i.e., orbital angular momentum (l), spin angular momentum (s) and corresponding moments are all quantized vectors. Hence the model is called as the vector atom model.

* The two concepts of vector atom model have been experimentally verified by Stern & Gerlach while studying the behaviour of atoms in non – homogeneous magnetic fields.

❖ Stern – Gerlach experiment (Verification of space quantization concept and electron spin):

Construction:

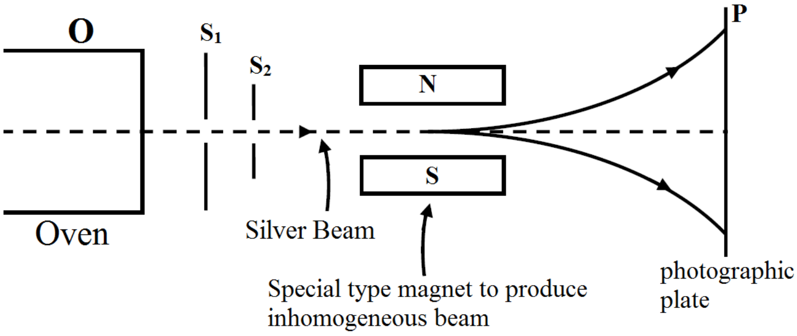
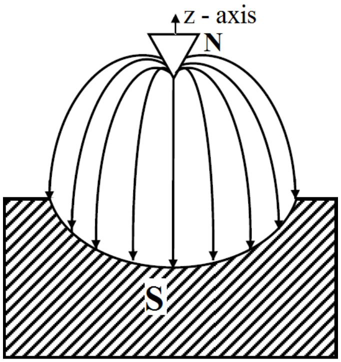


Fig 1.5



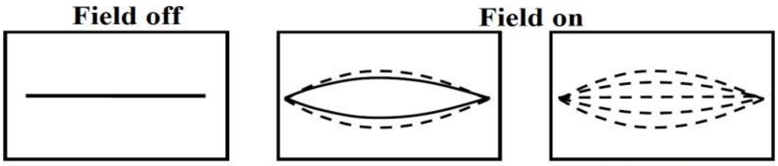
Side view of the magnet with lines of force

Fig 1.6

The experimental setup of Stern – Gerlach experiment is shown in fig. A beam of silver atoms is produced by heating silver in a small electric oven O. The slits produce a sharp parallel pencil beam of silver atoms. This beam is passed through an inhomogeneous magnetic field produced by pole pieces N & S of an electromagnet. The beam emerging out of the pole pieces strike a photographic plate P. The entire apparatus is housed in a highly evacuated glass vessel.

Working:

In the absence of magnetic field, a trace in the form of a narrow strip is obtained on the photographic plate P as shown in fig (a). When the inhomogeneous magnetic field is switched on, the strip splits up into two components as shown in fig (b).



(a) No field pattern (b) Actual pattern (c) Classical pattern

Fig 1.7

The splitting of silver beam into two components in inhomogeneous magnetic field verifies the electron spin and space quantization as explained below. Silver has an atomic no – 47 silver is monovalent element and 5s electron is responsible for its magnetic moment. When the silver atom is subjected to an inhomogeneous magnetic field, the two poles of the atomic magnet experience unequal force and hence there will be a transverse

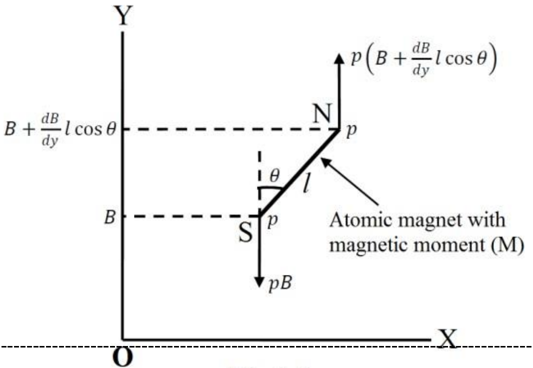


Fig 1.8

displacement of atom, so the silver atoms are deviated in transverse direction.

Theory:

The amount of transverse deviation of silver atoms can be calculated as follows.

Let the magnetic field is inhomogeneous in $y-$ direction with field gradient $\frac{dB}{dy}$. Let the atomic magnetic moment M, pole strength P and length L is inclined at angle θ with the field direction.

Let the field strength at one pole $= B$

Field strength at other pole $= B + \frac{dB}{dy}$

Force on one pole $= PB$

Force on other pole $= P\left(B + \frac{dB}{dy} l \cos\theta\right)$

Net force on silver atomic magnet $F_y = P\left(B + \frac{dB}{dy} l \cos\theta - B\right)$

$$= Pl\cos\theta \frac{dB}{dy} = M\cos\theta \frac{dB}{dy}$$

Due to this force, silver atoms are deviated transversely in inhomogeneous magnetic field with velocity v and leave the field of path length L in time t.

Deviation $= \frac{1}{2}\alpha_y t^2$

$\alpha_y = \frac{F_y}{m}$ and $t = \frac{L}{v}$

$dy = \frac{1}{2}\frac{M\cos\theta}{m}\frac{dB}{dy}\left(\frac{L}{v}\right)^2$

Theoretically dy is determined from above equation experimentally dy is determined from the maximum separation between the two traces obtained in the experiment. The theoretically as well as experimentally are in good agreement. This strongly supports the postulate of space quantization and the existence of electron spin.

❖ Quantum numbers associated with vector atom model:

The quantum numbers associated with each of electrons in an atom are

1. **The principal (or) total quantum number (n):**
   - This quantum number belongs to the principal orbit to which the electron belongs.
   - The principal (or) total quantum number n can have only non – zero positive integral values i.e., $n = 1, 2, 3, \dots \dots \dots \infty$. The energy levels (or) shells of electrons corresponding to $n = 1, 2, 3, \dots \dots \dots \infty$ are denoted by K, L, M, ...
   - The maximum number of electrons in a shell is $2n^2$.
   - In terms of principal quantum number n, the energy of the electron and its distance from the nucleus are $E_n = -\frac{13.6}{n^2}$ eV, $r_n = 0.259n^2$ A^0
2. **Orbital quantum number (L):**
   - This quantum number defines the shape of the orbital and orbital angular momentum of the electron.
     $L = \frac{lh}{2\pi}$ where $l = 0, 1, 2, \dots\dots\dots\dots (n-1)$
   - This quantum number is similar to the azimuthal quantum number n_ϕ of the Sommerfeld model.
     $l = 0$ corresponds to $s -$ orbital
     $l = 1$ corresponds to $p -$ orbital
     $l = 2$ corresponds to $d -$ orbital
     $l = 3$ corresponds to $f -$ orbital
3. **Spin quantum number (S):**
   - In vector atom model, an electron spins around its own axis and the spin angular momentum is given by
     $S = \frac{sh}{2\pi}$ where $s = \pm\frac{1}{2}$
   - Two electrons with same sign of spin quantum number $(s = \frac{1}{2})$ are said to have parallel spins while the others having opposite spins.
4. **Total angular momentum vector (J):**
   - The resultant angular momentum of the electron due to both orbital and spin motions is given by
     $J = \frac{jh}{2\pi}$ where $j -$ total angular momentum
     $j = l + s = l \pm \frac{1}{2}$
     $j = l + \frac{1}{2}$ where $\vec{l}$ & $\vec{s}$ are parallel.
     $j = l - \frac{1}{2}$ where $\vec{l}$ & $\vec{s}$ are anti parallel.
   - When the atom is placed in an external magnetic field, the magnetic field influences both the orbital and spin motions. So the directions of $\vec{l}$ & $\vec{s}$ is to be quantized. This gives rise to 3 more additional quantum numbers as follows.
5. **Magnetic orbital quantum number (m_s):**

Due to spin quantization, the orbital angular momentum $(\bar{L})$ can have orientations, such that its projections on magnetic field direction can vary from $-l\hbar$ to $+l\hbar$ in steps of one.

"The projection of $\bar{L}$ on the field direction is known as magnetic orbital quantum number".

$m_l = -l$ to $+l$ in steps of one.

Ex: for $l = 3, m_l = -3, -2, -1, 0, 1, 2, 3$

Number of values of $m_l = (2l + 1)$

Number of allowed orientations of $\bar{L} = (2l + 1)$

6. **Magnetic spin quantum number (m_s):**

   In an external magnetic field, the spin angular momentum S can have only two possible orientations for which the projections will be having values $m_s = -\frac{1}{2}$ & $+\frac{1}{2}$ either of these two values specifies the magnetic spin quantum number

   $\therefore m_s = -\frac{1}{2}$ (or) $+\frac{1}{2}$

7. **Magnetic total angular momentum number (m_j):**

   The value of m_j is the projection of the total angular momentum vector j on the field direction. Since $'j'$ can have half integral values, m_j can also assume half integral values.

   The possible values of m_j are $m_j = -j$ to $+j$ in steps of one.

   $\therefore$ Permitted values of $m_j = 2j + 1$

   Out of 7 quantum numbers given above, it is only the 4 quantum numbers n, l, m_l and m_s exclusively define the state of electron according to vector atom mode.

- **Coupling schemes**:

Generally two types of coupling known as Russell – Sander's coupling or L – S coupling and J – J coupling occurs between the orbital and spin angular momenta.

1. **L – S coupling**:

   This type of coupling occurs most frequently and hence known as normal coupling. In this coupling, all the orbital angular momentum vectors $\vec{l}$ of the electrons combine to form a resultant vector $\bar{L}$ and all the spin angular momentum vectors $\vec{s}$ likewise combine to form a resultant vector $\vec{S}$.

   Now the vectors $\bar{L}$ & $\vec{S}$ combine to form a vector $\vec{J}$ which represents the total angular momentum of the atom.

   $\bar{L} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \cdots$

   $\vec{S} = \vec{s}_1 + \vec{s}_2 + \vec{s}_3 + \cdots$

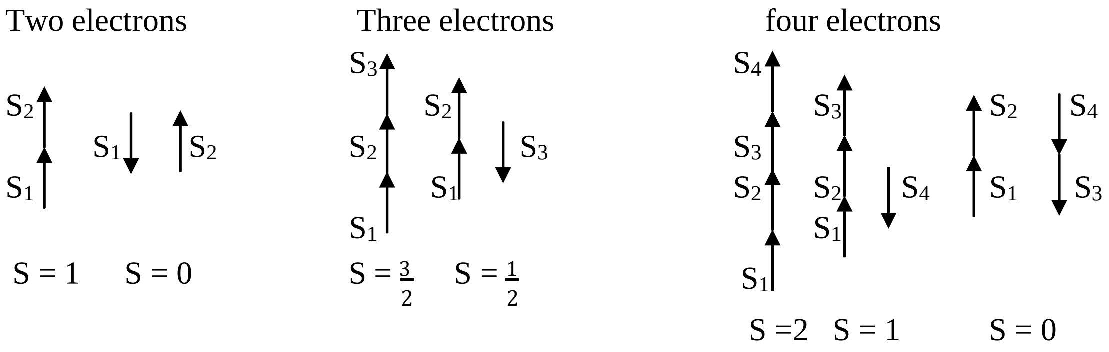
   $\vec{J} = \bar{L} + \vec{S}$

   Examples:

   For $3p$ and $3d$ electrons, $l_1 = 1, l_2 = 2$

The maximum value of $L = 2 + 1 = 3$
The minimum value of $L = 2 - 1 = 1$
∴ Possible values of L are 1, 2, 3

For $2p\ 3p\ 4d$ electrons
The maximum value of $L = 2 + 1 + 1 = 4$
The minimum value of $L = 2 - 1 - 1 = 0$
∴ Possible values of L are 0, 1, 2, 3, 4
So, L is always an integer i.e., 0, 1, 2, 3, 4.... the values of S depends upon the number of electrons and the direction of their spin vectors as given below.



From the above, we say that S is an integer for even number of electrons and half integer for odd number of electrons.
$J = integer$ where $S = integer$ i.e., for even number of electrons
$J = half\ integer$ where $S = half\ integer$ i.e., for odd number of electrons.
Note that J is always $+ve$ and never $-ve$ because it represents the total angular momentum of the atom.

2. **$J - J$ coupling**:

Sometimes it is observed that the interaction between the spin and orbital vectors in each electron is stronger than the interaction between either the spin vector (or) the orbital vectors of different electrons. In each case $J - J$ coupling is most suitable than $L - S$ coupling.

In $J - J$ coupling each electron is considered separately and its total angular momentum J is obtained by $j = l + s$, then the total angular momentum J of the atom would be vector sum of all the individual j vectors of the electrons. Thus

$j_1 = l_1 + s_1$, $j_2 = l_2 + s_2$, $j_3 = l_3 + s_3$
$J = j_1 + j_2 + j_3 + \cdots = \sum j_i$

- **Zeeman effect**:

The splitting of spectral lines when placed in a magnetic field is called Zeeman effect.

* If the magnetic field is very strong, each spectral line splits into 2 components in the longitudinal view and into 3 components in transverse view. This is known as Normal Zeeman effect.

* If the magnetic field is comparatively weak, each line splits into more than 3 components. This is known as anomalous Zeeman effect.

Experimental arrangement:

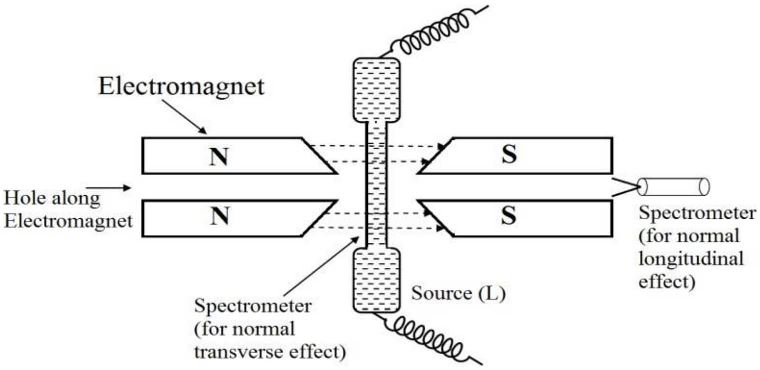


Fig 1.9

Zeeman apparatus consists of an electromagnet having conical polepieces and holes drilled along the length so that light from the source can pass through it.

The spectral lines are observed with a high resolution spectrometer.

When the source of spectral lines is placed in a strong magnetic field, the following features are observed.

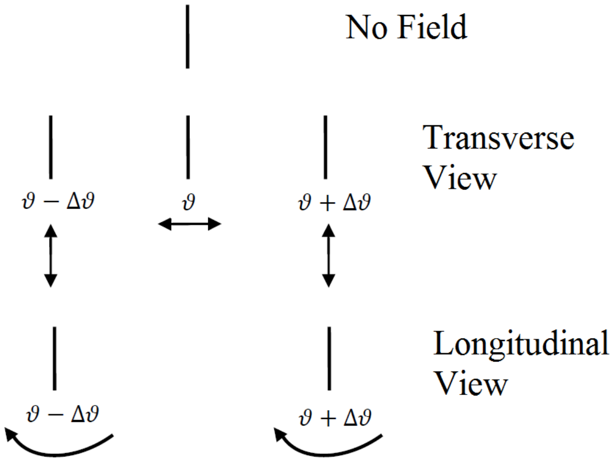


Fig 1.10

1. When the source is viewed at right angles to magnetic field it is found that the original line of frequency P splits symmetrically into 3 components of frequencies $P + \Delta P$, P, $P - \Delta P$ as shown in fig. It is observed that the original line is linearly polarised parallel to the magnetic field, where as the component lines on either side are polarised at right angles to the magnetic field.
2. When the source is view along the direction of the magnetic field, the same two additional lines with frequencies $P + \Delta P$ and $P - \Delta P$ are observed while the central line having frequency P is missing. These two lines are circularly polarised one being clockwise and other anti clockwise.

❖ **<u>Raman effect</u>**:

In 1928, Sir C.V Raman observed that the scattered light by a substance consists of greater and smaller frequencies in addition to the frequency of incident light. This is known as Raman effect.

The spectrum of scattered light is called Raman spectrum and the new lines are known as Raman lines.

1. The lines of greater frequency (small wave length) are called as anti stokes lines.
2. The lines of smaller frequency (greater wave length) are called as Stokes lines.

3. The displacements of lines (Raman displacements) are characteristics of scattering substance.

Characteristics of Raman lines:

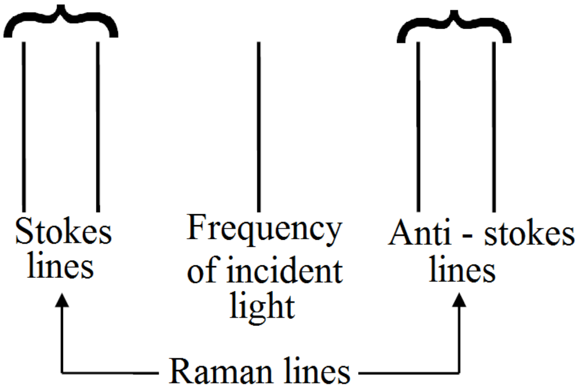


Fig 1.11

1. The frequencies of Raman lines depends on the frequency of incident light.
2. The displacement of Raman lines from the original line depend on the nature of scattering substance. On the other hand Raman displacements are independent of frequency of incident light.
3. The lines of greater frequency are called anti stokes lines, while the lines of smaller frequency are called stokes lines.
4. The Raman effect is purely a molecular phenomena.

Experimental arrangement of Raman effect:

The experimental arrangement for Raman effect is shown fig. It consists of (i) Source (ii) Raman tube (iii) Spectrograph.

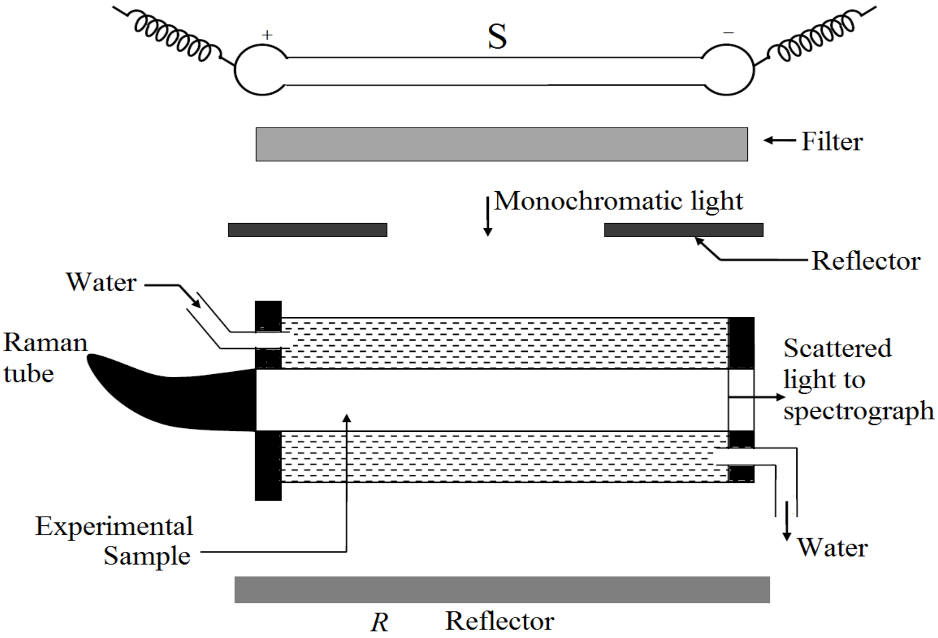


Fig 1.12

In fig, S is a mercury vapour lamp i.e., a source of light. The light is passing through a filter F to obtain a monochromatic beam, i.e., light of single frequency. This light is then allowed to pass through an opening in a metallic reflector and falls on Raman tube. Raman tube is a glass plate of about 1 to 2 cm diameter and 10 – 15 cm in length. One end of the tube has a flat glass surface and other end is drawn out into horn shaped and blackened outside. The tube is surrounded by a water jacket in which cold water is circulated to prevent overheating the sample in Raman tube.

The experimental sample is placed inside Raman tube. The scattered beam emerge from the flat end of the Raman tube and is examined by means of spectrograph.

Precautions: The source of light should be very strong otherwise the Raman lines be of very low intensity.

❖ Quantum theory of Raman effect:

According to quantum theory, the source of light emit photon (light quantum) of energy $⍰P_0$ where P_0 is the frequency of light when such photon hits a molecule the following three things might be happen

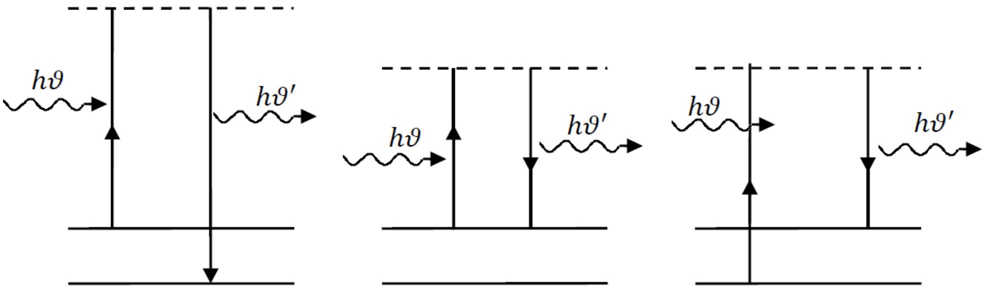
1. The molecule doesn't absorb energy from the photon but scatters it. In this case there will be unmodified line in the scattered beam.
2. The molecule may be in the excited state and when the photon hits the molecule, the molecule imparts some of its energy to photon. Now the energy of the molecule decreases. If E_1 and E_2 are initial and final energies of the molecule then decrease in the energy of the molecule $= E_1 - E_2$

   $\therefore$ energy of the photon $= ⍰P_0 + (E_1 - E_2)$

   The frequency of the scattered beam is given by

   $P_{as} = \frac{⍰P_0 + (E_1 - E_2)}{⍰} = P_0 + \frac{(E_1 - E_2)}{⍰}$

   i.e., the frequency of the scattered beam increases. This corresponds to the frequency of anti stokes line.



Anti - stokes lines $\lambda' < \lambda$

Rayleigh $\lambda' = \lambda$

Stokes lines $\lambda' > \lambda$

Fig 1.13

3. The molecule may absorb some energy from the photon. Let the energy of the molecule increases from E_2 to E_1. Then the energy of the photon decreases by $E_1 - E_2$

   $\therefore$ Energy of the photon $= ⍰P_0 - (E_1 - E_2)$

   $P_s = \frac{⍰P_0 - (E_1 - E_2)}{⍰} = P_0 - \frac{(E_1 - E_2)}{⍰}$

   i.e., the frequency of the scattered beam decreases. This corresponds to the frequency of stokes line. The 3 different situations are shown in figure.

❖ Applications of Raman effect:

1. To study the molecular structure of crystals and compounds.
2. To know the number of atoms in a molecule, their relative arrangement, relative masses and chemical bonds between them.

3. To study the composition in plastics, mixtures, etc.
4. To decide about single, double or triple bond.
5. To study the spin and statistics of nuclei.
6. To study the binding forces between the atoms or group of atoms in crystals.
7. To study the vibrational and rotational energy levels of homo – nuclear molecules,for example nitrogen, oxygen, etc.

- **Important Questions**:
  1. Explain the drawbacks or Limitations of Bohr's theory.
  2. Mention two important points of vector atom model.
  3. Explain L – S and J – J coupling schemes.
  4. What is Raman effect? Give its applications.
  5. Explain Sommerfeld atomic model and give relativistic correction.
  6. What is Zeeman effect? Explain Zeeman effect.
  7. Describe the Stern – Gerlach experiment.
  8. Explain various quantum numbers associated with vector atom model.
  9. What is Raman Effect? Describe the experimental setup to study the Raman effect. What are its applications?
  10. What is Raman Effect? Describe the experimental setup to study the Raman effect. Explain the quantum theory.

- **Problems**:
  1. The exciting line in an experiment 5460 A^0 and the stokes line is at 5520 A^0. Find the wavelength of anti – stokes line.
  2. A sample was excited by 4358 A^0 line. A Raman line was observed at 4447 A^0. Calculate the Raman shift. At what wavelength anti stokes line appears?

Unit – II

Chapter 2: Matter waves and Uncertainty principle

❖ de – Broglie hypothesis of matter waves:

Louis de – Broglie put forward a suggestion that like radiation, matter also exhibit wave nature. The following factors have lead de – Broglie to propose the concept of matter waves.

(1) Waves and
(2) The universe is composed of radiation and matter.
(3) The nature loves symmetry. Therefore, matter and radiation must be symmetrical.

If radiation can act as a wave sometimes and like a particle at other time, then material particles (electrons, protons etc) should act like a particle sometimes and like a wave at other time. The waves associated with material particles are called as matter waves.

Wave length of de – Broglie matter waves:

The expression of the wave length associated with material particle can be derived on the analogy of radiation as follows,

According to Planck's quantum theory, the energy of a photon is given by

$$E = hP = \frac{hc}{\lambda} \quad \ldots\ldots\ldots\ldots (1)$$

According to Einstein mass energy relation

$$E = mc^2 \quad \ldots\ldots\ldots\ldots (2)$$

From (1) and (2) $mc^2 = \frac{hc}{\lambda}$

$\lambda = \frac{h}{mc}$ where $mc -$ momentum associated with photon

If we consider the case of a material particle of mass m and moving with a velocity $'v'$, then the wave length associated with this particle is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$\boxed{\lambda = \frac{h}{p}}$$

Different cases:

Case (i): If E is the kinetic energy of the particle then $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$p = \sqrt{2mE}$

$\therefore$ de – Broglie wave length $\lambda = \frac{h}{\sqrt{2mE}}$

Case (ii): When a charged particle carrying a charge q is accelerated by a potential difference V_0, then its kinetic energy is given by $E = qV_0$

Hence, the de – Broglie wave length associated with this particle is

$$\lambda = \frac{h}{\sqrt{2mqV_0}}$$

Case (iii): when a material particle is in thermal equilibrium at a temperature T, then $E = \frac{3}{2}kT$, where $k -$ Boltzmann constant

So, the de – Broglie wavelength of a material particle at temperature T is given by

$$\lambda = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}} = \frac{h}{\sqrt{3mkT}}$$

Case (iv): de – Broglie wavelength of a electrons:

Let an electron of rest mass m_0 (9.1 × 10^{-31} kg) and charge q (1.6 × 10^{-19}C) is accelerated by potential difference of V volts. Then the de – Broglie wavelength of electrons is given by

$$\lambda = \frac{h}{\sqrt{2m_0qV}}$$

$$\lambda = \frac{6.625\times10^{-34}\,j-sec}{\sqrt{2\times(9.1\times10^{-31}\,kg)\times(1.6\times10^{-19}\,C)V}}$$

$$\lambda = \frac{12.26}{\sqrt{V}}A^0$$

❖ **Properties of matter waves**:

(1) The lighter the particle, the greater is the wavelength associated with it.

(2) The smaller is the velocity of the particle, the greater is the wavelength associated with it.

(3) When $v = 0, \lambda = \infty$ i.e., wave becomes undefined and if $v = \infty$ then $\lambda = 0$. This shows that the matter waves are generated by motion of particles.

(4) The velocity of matter wave depends on the velocity of matter particle, i.e., it is not constant while the velocity of electromagnetic wave is constant.

(5) The velocity of matter wave is greater than the velocity of light.

$w = \frac{c^2}{v}$, where $w =$ velocity of matter wave

$c -$ velocity of light

(6) The wave and particle behavior of moving bodies can never appear together in the same experiment.

❖ **Davisson and Germer electron diffraction experiment**:

The first experimental evidence of matter waves was given by Davisson and Germer. They succeeded in measuring the de – Broglie wavelength associated.

The experimental arrangement consists of an electron gun G where the electrons are produced and accelerated in the form of fine beam. The electron gun consist of tungsten filament f heated to dull red and electrons emitted

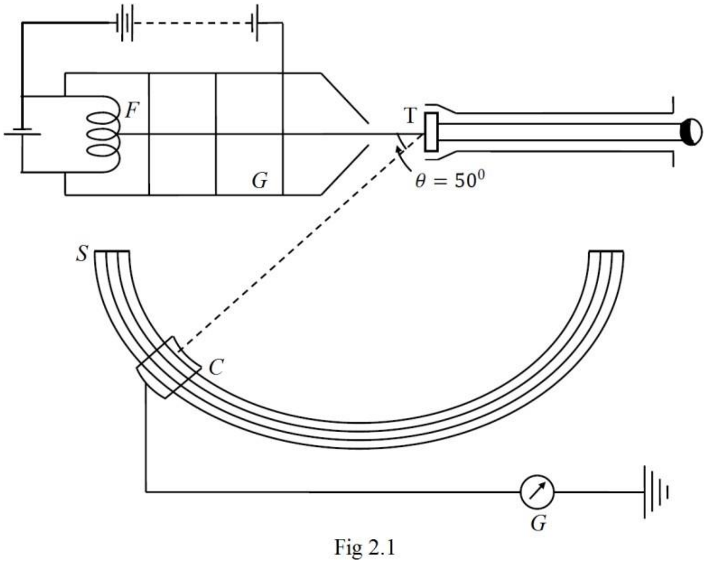


Fig 2.1

from it. The emitted electrons collimated and then accelerated by potential difference of V volt. The beam of electrons are made to fall on a nickel target. The electrons are diffracted on different targets and this is measured by Faraday's cylinder C which can moved on a circular scale.

Davisson and Germer measured the no of electrons diffracted using galvanometer G by varying θ and v. They observed a diffraction maximum for $\theta = 50^0$ and $V = 54$ volt
Under these conditions, the surface rows of atoms in nickel target acts like a diffraction grating producing the first order maximum for $\theta = 50^0$ and $V = 54$ volt
From nickel target the grating space $d = 0.91A^0$
From Bragg's law $n\lambda = 2d \sin\theta$
$\lambda = 2 \times 0.91 \times \sin 50^0$
$\lambda = 1.65A^0$ ………… (1)
From de – Broglie theory
$\lambda = \frac{12.26}{\sqrt{V}} A^0$
$\lambda = \frac{12.26}{\sqrt{54}}$
$\lambda = 1.67A^0$ …………. (2)
(1) and (2) are in good agreement with each other and hence the experiment confirms the existence of de – Broglie waves.

❖ Heisenberg uncertainty relation for x and p:

Statement: It is impossible to determine the position and momentum of a particle simultaneously with complete accuracy.

Explanation:

Case (i): Position is well defined, wavelength is very poorly defined. So, momentum is very poorly defined.

Case (ii): Position is well defined, wavelength is poorly defined. So, momentum is poorly defined.

Case (iii): Position is poorly defined, wavelength is well defined. So, momentum is well defined.

Case (iv): Position is very poorly defined, wavelength is very well defined. So, momentum is very well defined.

From fig (1) to fig (4)
The no of waves in the wave train associated with the particle increases. Hence the uncertainty in determining the position increases.

Expression:

From figure, we say that uncertainty in position Δx a n, where n is the no of waves.

$\Delta x = n\lambda$ ………….. (1)

Also fractional uncertainty in the λ is $\frac{\Delta\lambda}{\lambda} = \frac{1}{n}$, so $\frac{\Delta p}{p}$ a $\frac{1}{n}$

$\Delta p \approx \frac{p}{n}$ ………….. (2)

$\Delta x\, \Delta p = n\lambda \frac{p}{n}$

$\Delta x\, \Delta p \approx h$

Which is the Heisenberg uncertainty principle for position and momentum.

This gives only a qualitative result but if we observe fig (5), the uncertainty in determining both the position and momentum is high. So, the quantitative result becomes $\Delta x\, \Delta p \geq \hbar$

- **Time and energy uncertainty relation:**

We consider the time – energy uncertainty with the help of position – momentum uncertainty. Consider the case of every particle with rest mass m_0, moving with velocity v_x in $x-$ direction.

Kinetic energy $E = \frac{1}{2} m_0 v_x^2 = \frac{p_x^2}{2m_0}$

On differentiation $\Delta E = \frac{1}{2m_0} 2p_x \Delta p_x$

$\Delta E = \frac{p_x}{m_0} \Delta p_x$ …………. (1)

Where ΔE is the uncertainty in energy.

We know that $x = v_x \times t$

$\Delta x = v_x \Delta t$

$\Delta t = \frac{\Delta x}{v_x}$ …………. (2)

$(1) \times (2) \Rightarrow \Delta E \times \Delta t = \frac{p_x}{m_0} \Delta p_x \times \frac{\Delta x}{v_x}$ $p_x = m_0 v_x$

$\Delta E \Delta t = \Delta x\, \Delta p \approx h$

More accurate result is $\Delta E . \Delta t \geq \hbar$

- **Determination of position of electron through Gamma ray microscope**:

Consider the case of measurement of the position of the particle in the field of gamma ray microscope. The resolving power i.e., the smallest distance between the two points that can be just resolved by the microscope is

$\Delta x \approx \frac{\lambda}{2 \sin\theta}$ …………. (1)

Where λ is the wavelength of light and Δx is the uncertainty in determining the position of the particle.

When a photon of initial momentum $p = \frac{h}{\lambda}$ scatters the

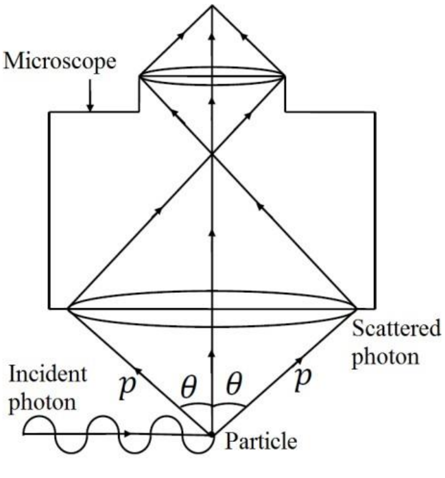


Fig 2.3

particle and enters the field of microscope. Thus its $x-$ component of momentum i.e., p_x may lie between $p\sin\theta$

and $-p\sin\theta$. The uncertainty in the $x-$ component of momentum is given by

$$\Delta p_x = p\sin\theta - (-p\sin\theta)$$

$$= 2p\sin\theta = 2\frac{h}{\lambda}\sin\theta \text{.......................................}(2)$$

From (1) & (2),

$$\Delta x\,\Delta p_x \approx \frac{\lambda}{2\sin\theta}\,2\frac{h}{\lambda}\sin\theta$$

$$\therefore \Delta x\,\Delta p_x \approx h$$

This shows that the product of uncertainty in position of momentum is of the order of Plank's constant.

- **<u>Diffraction by a single slit</u>**:

Consider a narrow beam of electrons passes through a slit and produces a diffraction pattern on the screen as shown in figure. The first minimum of the pattern is obtained by putting $n = 1$ in Bragg's law

$$n\lambda = 2d\sin\theta$$

$$\Delta y\,2\sin\theta = \lambda$$

Where Δy is the width of the slit and θ is the angle of deviation corresponding to first minimum.

The uncertainty in determining the position of electron is equal to the width Δy of the slit.

$$\Delta y = \frac{\lambda}{2\sin\theta} \qquad \text{...........} (1)$$

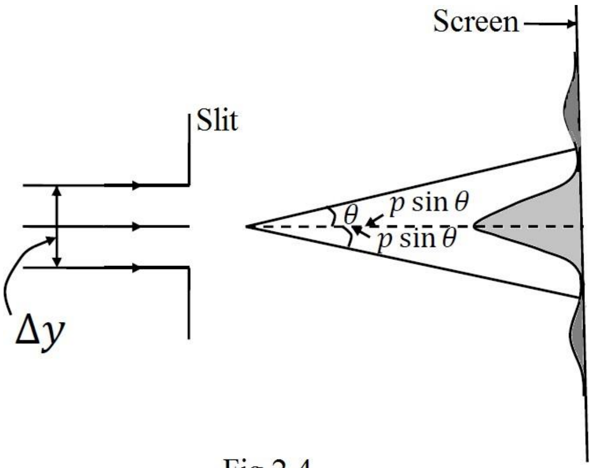


Fig 2.4

Initially the electrons are moving along $x-$ axis and hence they have no component of momentum along $y-$ axis. After diffraction of a slit they are deviated from their initial part and have a $y-$ component momentum $p\sin\theta$. The uncertainty in $y-$ component of momentum is

$$\Delta p_y = 2p\sin\theta = 2\frac{h}{\lambda}\sin\theta \text{........................}(2)$$

$$(1)\times(2) \Rightarrow \Delta y\,\Delta p_y = \frac{\lambda}{2\sin\theta}\,2\frac{h}{\lambda}\sin\theta$$

$$\therefore \Delta y\,\Delta p_y \approx h$$

- **Important Questions**:
  1. What are matter waves? Give their properties.
  2. What are matter waves? Write the expression for de – Broglie wavelength of matter waves.
  3. State and explain Heisenberg uncertainty principle.
  4. Derive the uncertainty relation for energy and time.
  5. Explain Davisson and Germer experiment on electron diffraction.
  6. State and explain Heisenberg uncertainty principle. Describe any experimental verification of it.

- **Problems**:
  1. Calculate the de – Broglie wavelength associated with the following:
     (i) A golf ball of 50g moving with a velocity of 20 m/sec.
     (ii) A proton moving with a velocity of 2200 m/sec.
     (iii) An electron moving with a kinetic energy of 50eV.
  2. Find the energy of the neutron in units of electron volt whose de – Broglie wavelength is 1 A^0.
     Given mass of the neutron = 1.674×10^{-27} kg
     Planck's constant $h = 6.60 \times 10^{-34}$ J – sec
  3. Energy of a particle at absolute temperature T is of the order of kT. Calculate the wavelength of thermal neutrons at 27^0 C.
     Given mass of the neutron = 1.674×10^{-27} kg
     Planck's constant $h = 6.60 \times 10^{-34}$ J – sec
     Boltzmann's constant = 8.6×10^{-5} eV/deg
  4. If the uncertainty in position of an electron is 4×10^{-10}m, calculate the uncertainty in its momentum.
  5. Using the uncertainty relation $\Delta E \cdot \Delta t \approx \frac{h}{2\pi}$, calculate the time required for the atomic system to retain the excitation energy for a line of wavelength 6000 A^0 and width 10^{-4} A^0.
  6. Assume that an electron is inside a nucleus of radius 10^{-15}m. Using uncertainty principle, estimate the K.E of the electron in eV.

Unit – III
Chapter 3: Quantum Mechanics

❖ Introduction:

In atomic systems, the classical mechanics fails to explain the microscopic system of particles due to uncertainty principle. According to uncertainty principle, the position and momentum of a particle cannot be measured accurately at the same time, the measurement of one quantity introduces an uncertainty into the other. Therefore, classical mechanics, which assumed both to have definite values at all instants, is not valid for atomic system. Thus, Bohr's model failed to give a complete explanation of the behaviour of atomic system.

According to de Broglie theory, a material particle is associated with a wave. So, a mathematical reformation using a wave function associated with matter wave is needed. Such a mathematical formation known as quantum mechanics (or) wave mechanics. It was developed by Schrodinger in 1926.

❖ Schrodinger time independent wave equation:

Schrodinger developed a wave equation for matter waves. It describes the wave nature of particle in mathematical form. Consider a group of de – Broglie waves associated with a moving particle. Let x, y, z be the wave displacement for the de – Broglie wave at any time. ψ is also called wave function.

The differential equation of a wave motion is given by

$$\frac{\partial^2\psi}{\partial t^2} = v^2 \left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right)$$

$$\frac{\partial^2\psi}{\partial t^2} = v^2\nabla^2\psi \qquad \text{............. (1)}$$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ (Laplacian operator)

The solution of equation is given by $\psi = \psi_0 \sin \omega t$ (or) $\psi = \psi_0 \sin 2\pi \mathrm{P} t$....................(2)

P − Frequency

$$\frac{d\psi}{dt} = 2\pi \mathrm{P}\psi_0 \cos 2\pi \mathrm{P} t$$

$$\frac{d^2\psi}{dt^2} = -4\pi^2\mathrm{P}^2\psi_0 \sin 2\pi \mathrm{P} t$$

$$\frac{\partial^2\psi}{\partial t^2} = -4\pi^2\mathrm{P}^2\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = -4\pi^2 \frac{v^2}{\lambda^2}\psi \qquad \text{............. (3)}$$

Substituting the value of $\frac{\partial^2\psi}{\partial t^2}$ from (3) in (1),

$$v^2\nabla^2\psi = -4\pi^2 \frac{v^2}{\lambda^2}\psi$$

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \qquad \text{............. (4)}$$

Now from de – Broglie relation, $\lambda = \frac{h}{mv}$

$$\nabla^2\psi + \frac{4\pi^2}{h^2} m^2v^2\psi = 0 \qquad \text{............. (5)}$$

But we know that total energy of the particle is given by E = PE + KE

$$E = \frac{1}{2}mv^2 + V$$

$\frac{1}{2}mv^2 = E - V$

$m^2v^2 = 2m(E - V)$

Substitute this equation in equation (5)

$\nabla^2\psi + \frac{4\pi^2}{h^2}2m(E - V)\psi = 0$

$\nabla^2\psi + \frac{8\pi^2 m(E-V)}{h^2}\psi = 0$

This is known as Schrodinger time independent wave equation.

$\nabla^2\psi + \frac{8\pi^2 m(E-V)}{\left(\frac{h}{2\pi}\right)^2 4\pi^2}\psi = 0$

$$\boxed{\nabla^2\psi + \frac{2m(E - V)}{\hbar^2}\psi = 0}$$

The above equation can also be expressed in following way

$\frac{\hbar^2}{2m}\nabla^2\psi + (E - V)\psi = 0$

$E\psi = (-\frac{\hbar^2}{2m}\nabla^2 + V)\,\psi$

$\hat{H}\psi = E\psi$

$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$ is known as Hamiltonian operator.

- **Schrodinger time dependent wave equation**:

To obtain Schrodinger time dependent wave equation, we make use the time independent wave equation given by

$\nabla^2\psi + \frac{2m(E-V)}{\hbar^2}\psi = 0$ .... (1)

The wave function ψ including time t can be written as $\psi = \psi_0 e^{-i\omega t}$·························(2)

Differentiating equation (2) with respect to time t,

$\frac{\partial\psi}{\partial t} = \psi_0(-i\omega)e^{-i\omega t}$

$= -2\pi \mathrm{P} i\psi$

$= -2\pi i\,(E/h)\psi$

$\frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar}\psi$

$E\psi = i\hbar\frac{\partial\psi}{\partial t}$

Substituting the value of $E\psi$ in Schrodinger time independent wave equation, we get

$\nabla^2\psi + \frac{2m}{\hbar^2}(i\hbar\frac{\partial\psi}{\partial t} - V\psi) = 0$

$\nabla^2\psi = -\frac{2m}{\hbar^2}(i\hbar\frac{\partial\psi}{\partial t} - V\psi)$

$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$

This is known as time – dependent wave equation.

$(-\frac{\hbar^2}{2m}\nabla^2 + V)\,\psi = i\hbar\,\frac{\partial}{\partial t}\psi$

$\hat{H}\psi = \hat{E}\psi$

$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$ Hamiltonian operator

$\hat{E} = i\hbar\frac{\partial}{\partial t}$ energy operator

❖ Physical significance of wave function:

1. The variable quantity which characterises de – Broglie wave is called as wave function ψ.
2. If ψ is the amplitude of matter waves at any point in space, then the particle density at that point may be taken as proportional to ψ^2. Hence ψ^2 is a measure of particle density.
3. Max born suggested a new physical significance to ψ which is generally accepted at present. According to Born $|\psi|^2$ not a measure of finding the particle density but it gives the probability of finding the particle at a point in a given time. More exactly, the probability of finding a particle in a small volume $dx\ dy\ dz$ is $\psi^2 dxdydz$.
   We know that total probability $= 1$
   $$|\psi|^2\ dxdydz = 1$$
   Which is known as normalization of wave function.
4. The probability zero corresponds to the certainty of not finding the particle somewhere in the space, i.e.,
   $$|\psi|^2\ dxdydz = 0$$

❖ Properties of wave function:

1. **ψ must be finite everywhere**

   If ψ is infinite at a point, then probability of finding the particle is infinite at that point. This is not possible. Hence ψ must have a finite or zero value at any point.
2. **ψ must be single valued**

   If ψ has more than one value at a point, it means that there is more than one value of probability for finding the particle at a point. This is obviously ridiculous.
3. **ψ must be continuous**

   For Schrodinger equation $\frac{d^2\psi}{dx^2}$ must be finite everywhere. This is possible only when $\frac{d\psi}{dx}$ has no discontinuity at any boundary where potential changes. This implies that ψ must be continuous.

❖ Postulates of quantum mechanics:

Postulate 1*: There is an associated wave function ψ with the system and any state of the system is described as fully as possible by this wave function*.

For a wave function ψ to describe any physical system, the following boundary conditions must be satisfied:

(1) $\psi(x)$ as well as $\frac{d\psi}{dx}$ must be finite for all values of x.

(2) $\psi(x)$ as well as $\frac{d\psi}{dx}$ must be continuous for all values of x in the region.

(3) $\psi(x)$ as well as $\frac{d\psi}{dx}$ must be single valued for all x in the region.

Postulate 2: ***With every dynamical variable, there is associated an operator.***

Classical quantity	**Quantum mechanical operator**
Cartesian components of position x, y, z	$\hat{x}, \hat{y}, z^\wedge$
Position vector $\vec{r}$	$r^\wedge$
Momentum $\bar{P}$	$-i\hbar\bar{\nabla}$
Cartesian components of linear momentum p_x, p_y, p_z	$(-i\hbar\frac{\partial}{\partial x}, -i\hbar\frac{\partial}{\partial y}, -i\hbar\frac{\partial}{\partial z})$
Total energy	$i\hbar\frac{\partial}{\partial t}$

Postulate 3: It is defined as the average of the result of a large number of measurements on independent systems.

The average value (or) expectation value of an observable quantity corresponding to operator O of a physical system in the state $\psi(x, y, z)$ is given by

$$(O) = \frac{\int \psi^* \hat{O} \psi \, dxdydz}{\int \psi^* \psi \, dxdydz}$$

- **Applications of Schrodinger wave equation**:
- **Particle in a one – dimensional box (or) particle in a 1 – D infinite potential well**:

Consider a particle of mass m in a 1 – D box extending from $x = 0$ to $x = a$. The width of the box is $'a'$. The particle is free to move along $x -$ axis between the walls A and B of the box.

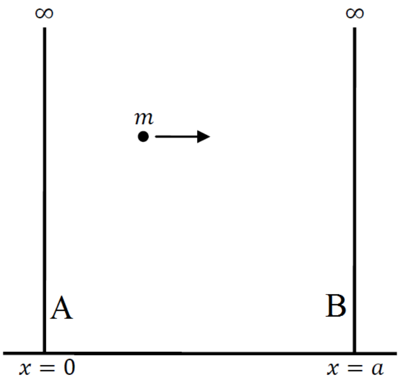


Fig 3.1

The potential energy of the particle $V = 0$

The Schrodinger equation $\nabla^2\psi + \frac{2m(E-V)}{\hbar^2}\psi = 0$ reduces for 1 – D box to

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0 \qquad \text{............ (1)} \qquad (\because V = 0)$$

Let $\frac{2mE}{\hbar^2} = k^2$ ....(2)

$$\therefore (1) \Rightarrow \frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0 \qquad \text{............ (3)}$$

The solution of this equation is given by $\psi = A\sin kx + B\cos kx$ ....(4)

Where A and B are constants.

As the walls of the box are of infinitely height at $x = 0$ and $x = a$, the particle can't escape from the box. So at boundary $(x = 0 \;\&\; x = a)$ ψ vanishes.

i.e., $\psi = 0$ at $x = 0$ .... (i)

$\psi = 0$ at $x = a$............................(ii)

Applying boundary condition (i) on equation (4), $B = 0$

$\therefore \psi = A\sin kx$ from equation (4)

Now applying boundary condition (ii) on above equation $0 = A\sin ka$

$A = 0$ or $\sin ka = 0$

But $A \neq 0$, $\sin ka = 0$

$ka = n\pi$

$k = \frac{n\pi}{a}$ .... (5) where

$n = 1, 2, 3, \ldots..$

The wave equation becomes $\psi = A\sin\frac{n\pi x}{a}$ .... (6)

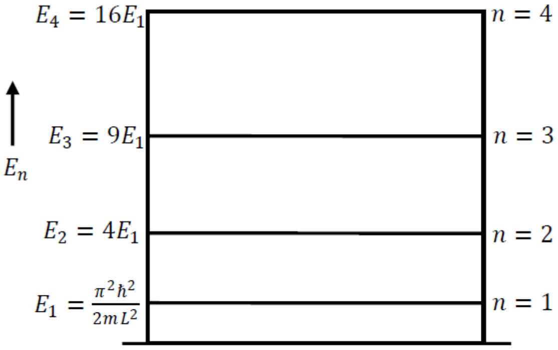


Fig 3.2

From equation (2) $\frac{2mE}{\hbar^2} = k^2$

From equation (5) $k^2 = \frac{n^2\pi^2}{a^2}$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{n^2h^2}{8ma^2} \qquad \text{........... (7)}$$

From equation (7), it is clear that inside the box the particle can only have discrete values of energy i.e., the energy of the particle is quantized energy levels are shown in figure.

Wave functions:

The constant of equation (6) can be obtained by applying this normalization condition

$$\int_0^a |\psi(x)|^2\,dx = 1$$

$$\int_0^a A^2 sin^2\frac{n\pi x}{a}dx = 1$$

a

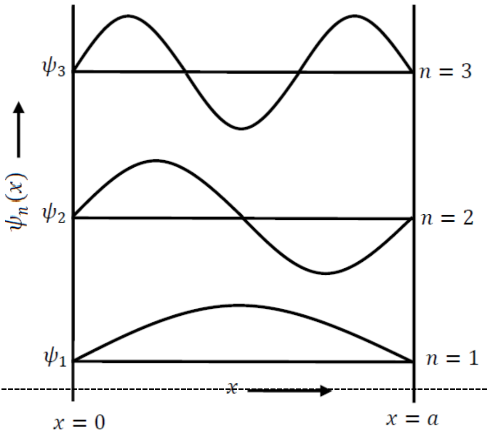


Fig 3.3

$$A^2 \int_0 [1 - \cos\frac{2n\pi x}{a}]\, dx = 1$$

$$\frac{A^2}{2}\left[x|_0^a - \frac{a}{2n\pi}\sin\frac{2n\pi x}{a}|_0^a\right] = 1$$

$$\frac{A^2}{2}(a) = 1 \Rightarrow A = \sqrt{\frac{2}{a}}$$

$$\boxed{\psi(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}}$$

- **Important Questions**:
  1. Discuss the basic postulates of quantum mechanics.
  2. Explain the physical significance of wave function.
  3. Derive the Schrodinger time independent wave equation.
  4. Derive the Schrodinger time dependent wave equation.
  5. Derive the energy Eigen values and normalized wave functions for a particle in one dimensional infinite square box.
  6. State the properties of wave function.

- **Problems**:
  1. What is the lowest energy that a neutron (mass of neutron = 1.67×10^{-27} kg) can have if confined to move along the edge of an impenetrable box of length 10^{-4} m?
  2. Calculate the energy difference between the ground state and the first excited state for an electron in a one dimensional rigid box of length 10^{-8} cm. (mass of electron = 9.1×10^{-31} kg and $h = 6.626 \times 10^{-34}$ j – s)
  3. Find the least energy of an electron moving in an infinitely high potential box of width 1 A^0, given mass of the electron 9.11×10^{-31} kg and $h = 6.626 \times 10^{-34}$ j – s

Unit – IV

Chapter – 4: General Properties of Nuclei

❖ Basic properties of Nucleus:

Rutherford α- particle scattering experiment concluded that atom of any element has a central core called nucleus and electrons are revolving around it. The basic properties of nucleus are

1. Nuclear mass
2. Nuclear charge
3. Nuclear radius
4. Nuclear density
5. Nuclear quantum states
6. Nuclear spin
7. Magnetic dipole moment of nuclei (μ_B)
8. Electric quadrupole moment (Q)

1. Nuclear mass: The mass of the nucleus is the sum of the masses of the neutrons (N) and protons (Z) contained in it.

Mass of nucleus = $Zm_p + Nm_n$

= $Zm_p + (A - Z)m_n$

* Usually nuclear mass is expressed in terms of atomic mass unit (a.m.u).

* One a.m.u = $1.66 \times 10^{-27} kg = 931.48$ MeV

Example: Carbon ($^{12}_{6}C$) has mass of 12 a.m.u

2. Nuclear charge: The charge of the nucleus is due to the protons contained in it.

Nuclear charge = Ze where $e = 1.6 \times 10^{-19} C$

$Z =$ proton number

Example: Carbon ($^{12}_{6}C$) nucleus has charge +6e

3. Nuclear radius: The volume of nucleus V is proportional to mass number A.

Since nucleus is spherical in shape $\frac{4}{3}\pi r^3 \propto A \Rightarrow r \propto \frac{3}{4\pi} A^{1/3}$

$r = r_0 A^{1/3}$ where $r_0 = 1.4 \times 10^{-15} m$ is a linear constant.

Example: for Carbon (A=12) $r = 1.4 \times 10^{-15} m \times (12)^{1/3} = 3.21 \times 10^{-15} m$

Uranium (A=238) $r = 1.4 \times 10^{-15} m \times (238)^{1/3} = 8.68 \times 10^{-15} m$

4. Nuclear density: Nuclear density of a nucleus is the ratio of mass of the nucleus to the volume of the nucleus.

Volume of nucleus $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = 14.15 \times 10^{-45} A$ metre^3

Mass of nucleus $M = A \times mass\ of\ proton = 1.673 \times 10^{-27} A\ kg$

Density of nucleus = $\frac{1.673 \times 10^{-27} A}{14.15 \times 10^{-45} A}$ kg/m^3

Density of nucleus = 1.16×10^{17} kg/m^3

5. Nuclear quantum states: The experiments on artificial radioactivity and $\alpha -$ and $\gamma -$ray spectra clear that every nucleus posses a set of quantum states (Energy levels). Transition between different nuclear states are accompanied by the emission of γ- rays.

6. **Nuclear Spin ($\boldsymbol{I}$):** Both the proton and neutron have an intrinsic angular momentum, commonly referred to as its spin. The magnitude of spin angular momentum is $\frac{1}{2}\hbar$. In addition, the nucleus posses orbital angular momentum due to motion about the centre of the nucleus. The resultant angular momentum of the nucleus is equal to the sum of spin and orbital angular momenta of all nucleons within the nucleus is referred to as the nuclear spin.

   ∴ Total angular momentum of nucleus $= \sqrt{I(I+1)}\hbar$

   The value of I depends upon the type of interaction between the nucleons.

7. **Magnetic dipole moment of Nuclei ($\boldsymbol{\mu_B}$):** we know that charged particle moving in a closed path is equivalent to a magnetic dipole. The spinning of e^- has an associated magnetic dipole moment of 1 Bohr magneton $\frac{e\hbar}{2m_e}$.

   i.e., $\mu_e = \frac{e\hbar}{2m_e}$ where e = charge of electron

   m_e = mass of electron

   Proton: Proton has a positive elementary charge (e). Due to its spin, it has magnetic dipole moment μ_N.

   $\mu_N = \frac{e\hbar}{2m_p}$ where e = charge of proton

   m_p = mass of proton

   Here μ_N is called nuclear magneton. Since $m_p = 1836 m_e$, So nuclear magneton is only (1/1836) times of Bohr magneton.

   Neutron: Neutron is a neutral particle. It was found that neutron has a magnetic moment $\mu_n = -1.9128\mu_N$.

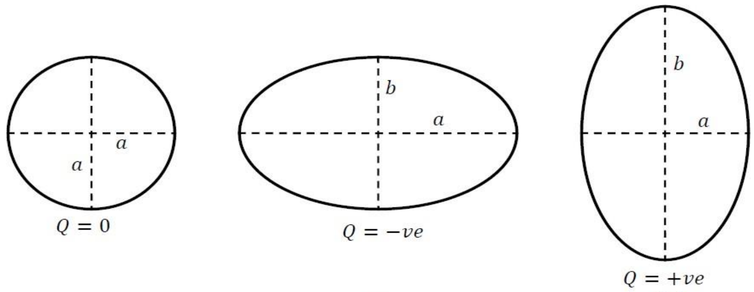
   To explain μ_n, it was assumed that neutron contain equal amount of +ve and -ve charges. If these charges are not uniformly distributed, then a spin magnetic moment may arise.

8. **Electric quadrupole moment ($\boldsymbol{Q}$):** Experiments showed that the shape of the nucleus not spherical but an ellipsoidal. *The deviation from the spherical symmetry is expressed in terms of a quantity known as electric quadrupole moment.*

   If the diameter along the axis of symmetry is $2a$ and diameter in a $\perp_{er}$ direction is $2b$, then the electric quadrupole moment is given by

   $Q = \frac{2}{5} Ze[b^2 - a^2]$ where Z = atomic number

   $Q = 0$, for spherical shaped nucleus
   $Q = -ve$, for oblate spheroid ($a > b$)
   $Q = -ve$, for prolate spheroid ($b > a$)



(a) (b) (c)

Fig 4.1

- **Binding energy of nucleus:** *"The binding energy is defined as the energy required to break the nucleus into its nucleons (protons and neutrons) is known as binding energy".*

When Z protons and $N = (A - Z)$ neutrons combine to form a nucleus, a certain mass (Δm) disappears. This is known as mass defect. This mass Δm is converted into energy and is called binding energy.

Binding energy (BE) = $\Delta m\, c^2$ where Δm = mass defect

c - velocity of light

Expression for Binding energy: If M is the experimentally determined mass of a nucleus having Z protons and N neutrons, then

$\Delta m = Zm_p + Nm_n - M$

$Zm_p + Nm_n =$ Calculated mass Where $m_p =$ mass of proton

$m_n =$ mass of neutron

This mass defect is converted into binding energy given by B.E. = $\Delta m\, c^2$

B.E. $= [(Zm_p + Nm_n) - M]c^2$

- If BE > 0, the nucleus is more stable and the energy must be supplied from outside to break the nucleus.
- If BE < 0, the nucleus is unstable and it will disintegrate by itself.

- **Binding energy of deuteron:**

There are two isotopes of ordinary hydrogen, one lighter $({}_1H^1)$ and other heavier $({}_1H^2)$. The heavier isotope of hydrogen contains 1 proton and one neutron in the nucleus. It is called heavy hydrogen or deuteron.

Now we will calculate the binding energy of deuteron in the following way

Mass of proton m_p = 1.007825 amu

Mass of neutron m_n = 1.008665 amu

∴ Total mass of proton and neutron = (1.007825+1.008665) amu = 2.016490 amu

Actual mass of deuteron = 2.014104 amu

∴ mass defect $\Delta m = (2.016490 - 2.014104)$ amu = 0.002386 amu

So, B.E. (E_B) = 0.002386 × 936 MeV = 2.23 MeV

This shows that when a deuteron is formed, 2.23 MeV of energy is released which is the binding energy of deuteron. Further to break the deuteron nucleus into proton and neutron, the minimum energy required is 2.23 MeV.

- **Models of nuclear structure:**

Liquid drop model: In liquid drop model, the forces acting in the nucleus are assumed to be similar to the molecular forces in a liquid drop. This model was proposed by Bohr who observed that there are certain similarities between nucleus and a liquid drop. The similarities are

1. The nucleus is supposed to be spherical in shape just like a liquid drop which is also spherical due to surface tension forces.
2. The force of surface tension acts on the surface of the liquid drop similarly there is a potential barrier at the surface of the nucleus.

3. The density of a liquid drop is independent of its volume similarly nuclear density is constant and is independent of its volume.
4. The intermolecular forces in a liquid drop are short range forces. One molecule in a liquid drop interact only their immediate neighbours similarly nuclear forces are short range forces. Nucleons also interact only with their immediate neighbours.
5. The molecules evaporate from a liquid drop on raising its temperature similarly when energy is given to a nucleus by bombarding it with a projectile a compound nucleus is formed which emits nuclear radiations immediately.
6. When a small liquid drop is allowed to oscillate, it breaks up into smaller drops. The process of nuclear fission is similar to this concept in which the nucleus breaks up into two smaller nuclei.

Semi – Empirical mass formula or Bethe – Weizsacker formula (Based on liquid drop model)

The liquid drop model can be used to obtain an expression for the binding energy of a nucleus. In 1935, Von Weizsacker obtain an expression for the BE of a nucleus. Later on this expression was modified by Bethe and others but the main outlines remains same. This modified expression is known as semi empirical mass formula.

According to this formula, the mass of a nucleus can be expressed by

$_{Z}X^{A} = Zm_p + Nm_n - E_b$, ……….. (1) where is the E_b binding energy in amu

The value of E_b is calculated empirically as made up of a number of correction terms given by

$E_b = E_v + E_s + E_c + E_r + E_p$ ………… (2)

Where E_v = volume energy correction

E_s = surface energy correction

E_c = Coulomb energy correction

E_r = asymmetric energy correction

E_p = pairing energy correction

(i) **Volume energy correction**: It has been experimentally verified that B.E of a nucleus is proportional to the total no of nucleons (A) in it.

i.e., E_v a A

$E_v = a_v A$ where a_v is a constant.

(ii) **Surface energy correction**: The nucleus has some nucleons on its surface. The nucleons on the surface of the nucleus are bind less tightly like the water molecules on the surface of the water. This gives to slight decrease in B.E of the nucleus. So, it has negative contribution.

We know that no of nucleons on the surface is proportional to surface area of nucleus.

No of nucleons on the surface a r^2

∴ Surface energy E_s a r^2

E_s a $(r_0 A^{1/3})^2$

$E_s = -a_s A^{2/3}$

(iii) **Coulomb energy correction**: Coulomb repulsion force between the protons in a nucleus causes a decrease in B.E.

∴ Coulomb energy $E_c \propto \frac{3}{5}\frac{q^2}{r}$

$$E_c \propto \frac{3}{5}\frac{(Ze)^2}{r_0 A^{1/3}}$$

$$E_c = \frac{3}{5}\frac{Z^2 e^2}{r_0 A^{1/3}}$$

$$E_c = -a_c \frac{Z^2}{A^{1/3}}$$

More accurately $E_c = -a_c \frac{Z(Z-1)}{A^{1/3}}$

(iv) **Asymmetry energy**: It has been observed that nuclei are most stable when nucleus contains equal number of protons and neutrons, *i.e.*,$N = Z$. This is called symmetric effect. As the value of A increases, the no of neutrons increases and hence B.E decreases. This decrement is known as symmetry energy correction.

Excess of neutrons = $N - Z$ = $(A - Z) - Z = A - 2Z$

∴ Asymmetry energy correction $E_a = -a_a \frac{(A-2Z)^2}{Z}$

(v) **pairing energy correction**: It has been observed that nuclei that containing even no of protons and neutrons are most stable and on the other hand, nuclei containing odd number of protons and neutrons are least stable. This pairing effect changes the B.E as shown below:

Number of protons Z	Number of neutrons N	E_b
Even	Even	$+a_p A^{-3/4}$
Odd	Even	0
Even	Odd	0
Odd	Odd	$-a_p A^{-3/4}$

From eq (2) $E_b = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{Z} + a_p A^{-3/4}$

From eq (1) ${}_Z^A\mathbf{X} = Zm_p + Nm_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} - a_p A^{-3/4}$

Which is the semi empirical mass formula.

Merits of liquid drop model:

(1) The liquid drop model explained many facts regarding nucleus such as observed B.E of nucleus and the stability against α, β disintegration and fission.

(2) The calculation of atomic mass and B.E can be done with good accuracy using liquid drop model.

Demerits of liquid drop model:

(1) This model failed to explain magic numbers.

(2) It failed to explain measured spins and magnetic moments of nuclei.

❖ <u>Nuclear Shell model</u>:

The shell model of the nucleus assumed that the energy structure of nucleus is similar to that of electrons outside the nucleus. According to shell model, nucleons are grouped in shells just like electron shell outside the nucleus. The shell model is also known as independent particle model.

Evidence for shell model:

It is observed that a nucleus is stable if it has a definite no of protons or neutrons or both. These numbers are known as magic numbers. These numbers are 2, 8, 20, 50, 82, 124, etc.

Observations in favour of magic numbers:

(1) If isotope of an element having isotopic abundance greater than 60% belong to magic number category.
(2) In nature, ${}_2He^4$ and ${}_8O^{16}$ are more stable and hence 2, 8 indicate stability.
(3) Tin (${}_{50}Sn$) and calcium (${}_{20}Ca$) has stable isotopes. So elements with Z = 20 and Z = 50 are more stable. So, numbers 20, 50 indicate stability.
(4) All the 3 main radioactive series decay to Pb^{208} with Z = 82 and N = 126. This indicates stability.
(5) It was found that some isotopes are spontaneous neutron emitters. These are ${}_8O^{17}$, ${}_{36}Kr^{87}$, ${}_{54}Xe^{137}$ for which N = 9, 51, 83 respectively which can be written as 8+1, 50+1, 82+1. If we take the loosely bound neutron as a valence neutron, the neutron numbers 8, 50 and 82 represents greater stability.
(6) It was found that the nuclei having no of neutrons equal to magic numbers can't capture a neutron because the shells are closed.

So, from the above conclusion, we say that magic numbers indicates stability which lead to the concept of nuclear shell model.

Merits:

(1) Shell model has been successful in explaining the magic numbers.
(2) Shell model explained the observed angular momenta, magnetic moments and electric quadrapole moments of nuclei.

Demerits:

(1) The shell model failed to explain large nuclear quadrapole moments and spherical shapes of many nuclei.

❖ **Important Questions**:

1. Describe the basic properties of nucleus.
2. Explain about the liquid drop model of nucleus of an atom. (Nov – 2016/10 marks)
3. Explain Binding energy of deuteron. (Nov – 2016/5 marks)
4. Explain the shell model of nucleus.
5. Explain the different terms contributing to nuclear binding energy and hence deduce semi – empirical mass formula.
6. Explain the terms (a) mass defect (b) Binding energy (c) Nuclear density (d) Magnetic dipole moment

❖ **Problems**:

1. What is the mass number A of the nucleus whose radius is $r = 2.71$ fermi? Given that $r_0 = 1.3 \times 10^{-15}$m.
2. Calculate the mass number of a nucleus whose radius r is 3.9×10^{-15} m. (Given that $r_0 = 1.3 \times 10^{-15}$m).
3. The radius of Ho^{165} is 7.731 fermi. Find the radius of He^4.
4. Find the binding energy of an α - particle from the following data:
   Mass of the helium nucleus = 4.001265 a.m.u
   Mass of proton = 1.007277 a.m.u
   Mass of neutron = 1.008666 a.m.u
   Take 1 a.m.u = 931.4812 MeV

Unit – V

Chapter – 7: Super conductivity

- **Super conductivity**: The phenomenon of super conductivity was discovered by Kammerlingh ones while he was measuring the resistivity of mercury at low temperature. He observed that the electrical resistivity of pure mercury passed into a new state called super conductivity state. The temperature at which the resistance disappears is called the transition temperature or critical temperature.

  Definition: The phenomenon showing complete disappearance of electrical resistance below a critical temperature T_c $(T_c = 4.2\ K\ for\ mercury)$ is called as superconductivity. Those substances showing this property are called as super conductors.

  Example:

  (1) Mercury $T_c = 4.2\ K$ (2) Niobium $T_c = 9.46\ K$
  (3) Aluminium $T_c = 1.19\ K$ (4) Lead $T_c = 7.19\ K$

  Basic experimental facts:

  Following are the basic experimental facts of super conducting materials.

  1. At room temperature, super conducting materials have greater resistivity than other elements
  2. The transition temperature is different for different isotopes of an element. It decreases with increasing atomic weights of isotopes.
  3. By adding impurities to the super conducting material, its critical temperature T_c can be lowered.
  4. There is no change in the crystal structure in super conducting state. This means that super conductivity is more concerned with the conduction electrons than the atoms.
  5. The superconductivity is characterized by zero electrical resistance. It can conduct electrical current even in the absence of an applied voltage also the current can persist for year without any detectable decay.
  6. The elastic properties don’t change due to the transition of superconducting state.
  7. All the thermo electric effects disappear in super conducting state.
  8. When a sufficient strong magnetic field is applied to a super conductor its super conducting property is destroyed.
  9. The entropy in a super conducting state is less when compared with normal state. So super conducting state is more orderly state than normal state.
  10. The thermal conductivity decreases when the substance transits to super conducting state.

- **Zero resistance – explanation of super conductivity**:

  The electrical resistance of ordinary metal is due to the collision between conduction electrons and ions of the crystal lattice. In super conducting state the electrons scatter in pairs rather than individual. This gives to an exchange force between electrons. When the electrons have opposite spins and momenta, there is a very strong force of repulsion. Thus all conduction electrons become a bound system. Now a transfer of energy takes place from this bound system to lattice ions.

When an electric field is applied to a substance in super conducting state, the pairs of electrons gain additional kinetic energy and produce an electric current. Since these electrons do not transfer any energy to the lattice they are not slowed down. It means that the substance does not possess any electrical resistivity. Hence the resistance of super conductor is zero.

❖ Effect of external magnetic field or temperature dependence of critical field:

Super conductivity can be destroyed if a sufficient strong magnetic field is applied. In other words, the super conducting materials restore its normal resistance when a strong magnetic field is applied to a super conductor in order to destroy its super conductivity is called as the critical magnetic field. If the applied magnetic field exceeds the critical value, the superconducting state is destroyed. The variation of critical magnetic field with temperature is shown in figure. From figure, it is clear that the normal conducting state is restored if the applied magnetic field is greater than the critical value or the temperature of the specimen is raised above critical temperature T_c. In other words, we can say that for the super conducting state to exist there must be a suitable combination of temperature and magnetic field.

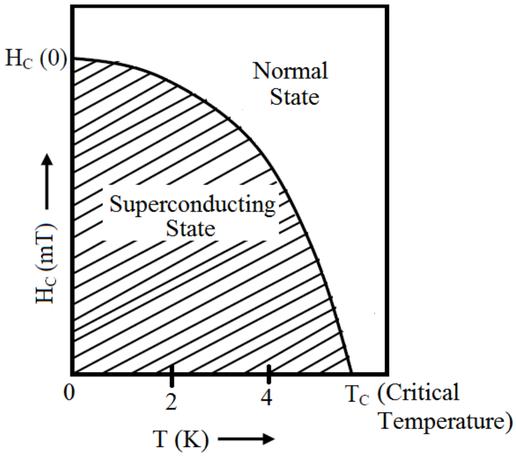


Fig 7.1

From the figure, the curve is nearly parabolic and hence it is represented by

$$H_c(T) = H_c(0)\,[1 - \frac{T^2}{T_c^2}]$$

Where $H_c(T)$ = Maximum, critical field at temperature T

$H_c(0)$ = Maximum, critical field at temperature 0^0 K

T_c – Critical temperature

❖ Meisner effect (flux exclusion):

When a super conductor is cooled in a magnetic field below critical temperature, then the lines of magnetic induction are expelled from the material. This effect is called as meisner effect.

Fig (a) shows the super conductor in normal state (i.e., $T > T_c$) and the magnetic lines of forces pass through it. But when the specimen is cooled below its transition temperature (fig b), the magnetic lines of forces are expelled out of the specimen.

The expulsion of magnetic lines of force from a super conducting material when it is cooled below the transition temperature in a magnetic field is called meisner effect.

Important points on meisner effect:

(1) Meisner effect is reversible when the temperature is increased above T_c , the flux suddenly penetrates through the specimen and the substance comes to its normal state.

(2) A super conductor is a perfect diamagnetic. The reason is that the magnetic field induction B in a super conductor is zero.

(3) The difference between a perfect conductor and super conductor is that the former is only an ideal conductor while the later is simultaneously an ideal conductor and an ideal magnet.

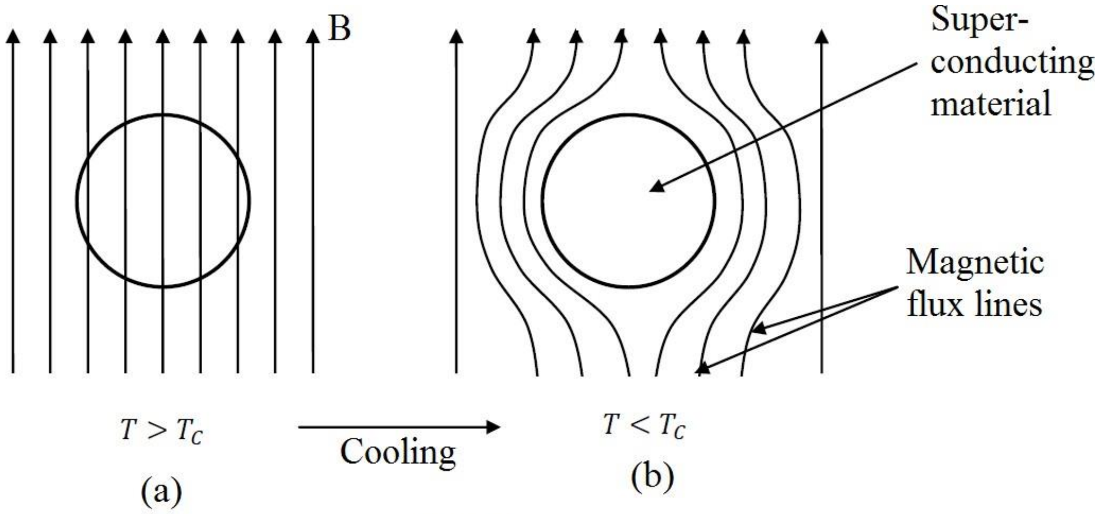


Fig 7.2

- **<u>Type – I and Type – II super conductors</u>**:

Based on magnetic behaviour, the super conductors are classified into following two categories.

1. Type – I super conductors or Soft super conductors
2. Type – II super conductors or Hard super conductors

<u>Type – I super conductors</u>:

The dependence of magnetization of super conductor of type – I as function of external field H is shown in fig. It is obvious from the figure that up to the critical field strength (H_c) . The magnetization of super conductor grows in proportion to the external field. As soon as the applied field exceeds H_c , the magnetization abruptly drops to zero.

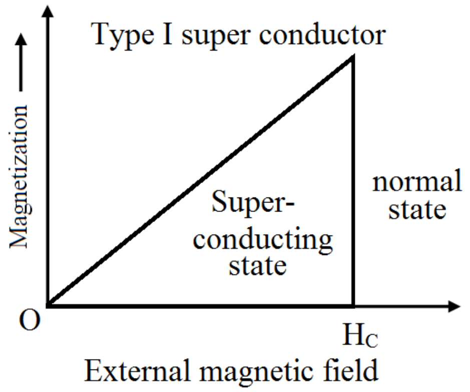


Fig 7.3

So, type – I super conductor is one in which the transition from super conducting state to normal state in presence of magnetic field occurs sharply at the critical value H_c.

In presence of an external magnetic field $H < H_c$, type – I super conductor in super conducting state is a perfect diamagnetic. When H exceeds H_c, the super conductor enters the normal state i.e., it loses its diamagnetic property completely. In this state, the magnetic flux penetrates throughout the super conductor.

In critical field, H_c value for type – I super conductor is found to be very low. Aluminium, lead and indium are examples of type – I super conductors.

Type – II super conductors: The type – II super conductor is characterized by two critical magnetic fields H_{c_1} and H_{c_2}.

The description of the curve is as follows:

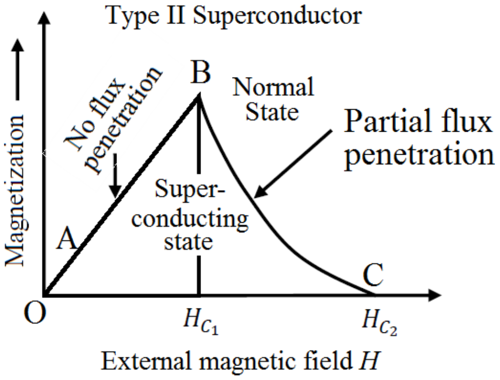


Fig 7.4

1. For the field strength below H_{c_1} , the super conductor expels the magnetic field from its body completely and behaves as a perfect diamagnetic H_{c_1} , is called the lower critical field. The curve is represented by AB.
2. As the magnetic lines increases from H_{c_1} , the magnetic field lines begin to penetrate the material, the penetration increases until H_{c_2} is reached. H_{c_2} is called the upper critical field. At H_{c_2} , the magnetization vanishes completely i.e., the external field has completely penetrated into super conductor and destroyed the super conductivity.

   In the region from H_{c_1} to H_{c_2} , the specimen assumes a complicated mixed structure of normal and super conducting states. The super conductor is said to be in a mixed state which is commonly known as vortex state in the region H_{c_1} and H_{c_2} , the material is in magnetically mixed but electrically it is a super conductor.
3. After H_{c_2} , the material turns to normal state. So, type – II super conductor is one which the characterized by two critical fields H_{c_1} and H_{c_2} and transition to normal state takes place gradually as magnetic field is increased from H_{c_1} to H_{c_2}.

   The most important advantage of type – II super conductor is the value of critical field H_{c_2}, which is many more times (even 100 times) higher than the value of H_{c_1} for type – I super conductor. So, the most important use of type – II super conductor is to build up a device which can work in high magnetic fields such as super conducting magnets.

- **Applications of super conductors**:

A super conductor find many practical applications. Few of them as follows

1. **Power transmission**: Electrical power transmission through any conductor is always accompanied by energy loss l^2R, where l is the current and R is the resistance of the conductor. If super conductors are used, the losses will be eliminated and power transmission can be done at a lower voltage level.
2. **Super conducting magnets**: As electromagnet made by using coils of super conducting wires or cables is called super conducting magnet. The main advantage is that once the current is setup, the conductor requires no source of emf to drive the current. Among many super conductors Niobium Titanium (Nb – Ti), a type – II super conductor is the common for super conducting magnets used for magnetic resonance imaging (MRI) technique employed to generate images of body cross section. This technique is much safer than using X – rays.

3. **Very strong magnetic fields**: Very strong magnetic fields (of the order of tesla by consuming only 10 kV) can be generated with coils made of super conducting materials. The cost of such magnets is quite lesser than conventional electromagnets. High magnetic fields are required in many areas of research and diagnostic equipments in medicine.
4. **SQUIDS** (super conducting quantum interference devices): Super conducting rings that act as storage devices for magnetic flux. They are used to detect very minute changes in the magnetic field of a human brain or body.
5. **For progress of computer technology**: At present, due to heat generated through I^2R losses there is a limit to which the components can be crowded on a chip of given size. The use of super conductors will make it possible to cram more circuits in a given area.

❖ **Important Questions**:

1. Explain type I and type II superconductors.
2. Write the applications of superconductors.
3. Explain Meissner effect.
4. Explain Isotope effect.